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On the determination of characteristic temperatures in power oil transformers during transient states

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Contents: Under usual assumptions the equations describing thermal phenomena in a transformer are established. Without neglecting any substantially important factor, algorithms are developed for predicting the temperatures in critical spots in non-stationary states occurring either in practice or in load/time diagrams. This paper suggests the determinaton of conductances dependent on temperature and constant capacitances of equivalent thermal circuits of a transformer. They are based on a small number of experiments (as simple as possible) and on a parameter estimation procedure. In such a way the temperatures of transformer parts in different transient states can be determined by computer simulation established on equivalent circuits, so the operators in power and sub-stations will be able to load their transformers close to the permitted limit. It is felt that this paper may give a stimulation for a possible revision of IEC rules.

Ermittlung charakteristischer Temperaturen von Leistungs-Öl-Transformatoren im nichtstationären Betrieb

Übersicht: Die Gleichungen des Temperaturverhaltens eines Transformators werden unter den üblichen Annahmen aufgestellt. Es werden Algorithmen entwickelt, welche die Vorausbestimmung der Temperatur an kritischen Stellen bei nichtstationärem Betrieb erlauben. Vorgeschlagen wird die Bestimmung der temperaturabhängigen und -unabhängigen Parameter auf Grund von wenigen und einfachen Messungen. Damit kann das Personal in Verteilerstationen in die Lage versetzt werden, mit Hilfe von PC's die Belastung der Transformatoren bis zu den zulässigen Grenztemperaturen auszudehnen. Die Autoren möchten damit einen Anstoß zu einer möglichen Überarbeitung von IEC-Vorschriften geben.

1 Introduction

The principal factor determining the life of a power transformer, as well as the safety margin in its everyday operation, which consists of time-varying load and dangerous states, is its temperature. Therefore, it is plausible why the interest in assessing the temperatures in critical points is so great. This refers to both stationary and transient temperatures. If one knows the temperature rise as the response to a certain load and other aforementioned influences, one would be able to predict them. In other words, the temperature can be calculated if the load, its duration and other factors are known.

The temperatures of critical spots in transient states cannot be calculated in an easy or quick manner. The reasons are numerous; let us mention only some of them. In the first place the material and geometry of a transformer are complex. Further, both the thermal parameters and winding power losses are strongly dependent on temperature level. Generally, the load varies erratically in time, as well as the ambient temperature which oscillates each day, each month and each year between broad limits. So, there are a lot of reasons why this problem is not yet satisfactorily solved.

The complexness of the transformer temperature prediction had been understood as early as the transformer came into practical use. In one of the fundamental papers written on this subject [1], Bach was the first who explicitly showed that there exist at least three different bodies making up a transformer; he developed a theory on transient thermal states of a system consisting of n different bodies, staying in the linear domain. Even in this simplified case, the solution of the differential equations has a clumsy and unpractical form, so that they have not found their application in practice. Even today, another procedure is most often used: when the oil temperature rise is considered, the transformer is supposed to be a thermally homogeneous linear body, and when temperature of the winding is studied, the copper part of the transformer is such a body, whereas its ambient temperature, i.e. that of the oil, is slowly varying in time. This procedure is accepted even in the international regulations [2, 3]. Obviously, due to the limited accuracy of temperatures so obtained, the transformer, loaded on the basis of them, could not be utilized up to the limits of its capability. Therefore, the authors of the present paper feel that every effort should be made to find out and develop methods, which would make it possible to calculate the critical temperatures much more accurately, providing a relative simplicity of application. It is felt that the development and spreading of personal computers (PC's) allows the successful solving of this task.

The aim of this paper is to set up the equations describing the thermal phenomena in the object, without neglecting any substantially important factor. Algorithms will be established for calculating the critical temperatures under different loading conditions. In such a way, conditions may be created which would eventually lead to a proposal for revision of IEC regulations [2, 3].

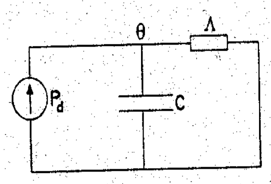

Fig. 1. Thermal equivalent circuit of a transformer as an isothermic body

2 Review of the most important bibliography

In the further text, the methods developed by various authors or international committees will be shortly exposed. Special attention will be drawn to typical restrictions and neglections applied in these papers [2–7].

When transient states are investigated, in the first approximation the single-body approach is often used, which can be illustrated by an equivalent circuit (Fig. 1) having a thermal capacitane C and a thermal conductance Λ, resulting in a single time constant ($T = C/\Lambda$). Transient phenomena are studied by means of the first-order transfer function, with power losses P_d as the input and the temperature rise (of the transformer as a whole) θ as the output.

It is interesting to note that this rough approximation is used even in standards and recommendations, but it eventually holds only for oil temperature calculation, not for that of the winding. The approach usually applied in regulations [2, 3, 7] (with a single time constant for the oil and the instantly changing copper temperature rise in respect to the oil temperature when a sudden load change occurs) could be acceptable if only those temperature changes were considered which take place after a certain elapse of time.

3 The three-dimensional and non-linear heat transfer

The heat transfer in the transformer is obviously three-dimensional. The most significant mode of heat transportation is the convection, which is non-linear in nature. Therefore, when both stationary and transient temperatures are investigated, in order to use standard solving techniques without complications, the model used is one-dimensional with constant parameters. It is valid only within a narrow temperature range. On the other hand, when large temperature variations are involved, the strong dependence of Λ on temperature must be taken into account. The specific heat, i.e. the thermal capacity, may be taken as constant when temperature is changing. The non-linearity is commonly treated throughout textbooks, papers and standards [2, 4, 5, 7, 8]. It is customary to use the following mathematical dependencies for expressing the non-linearity in oil convection

$$\Lambda = K\theta^{0.25} \tag{1}$$

or

$$\Lambda = \Lambda_0(1 + \beta\theta) \tag{2}$$

where the meaning of designations is obvious.

As to the three-dimensional character of the temperature field, it is considered very rarely. Let us mention the hot spot temperature, which is calculated by means of simple empiric or semi-empiric formulas [2, 3].

4 Mathematical model

The basis for the development of the mathematical model are the following assumptions.

(A) The transformer consists of three (sometimes only two) bodies, each being thermally homogeneous and having one single temperature. These are the temperatures of (1) the winding (the upper part, obtained as the sum of average copper temperature and a half of the difference between oil temperatures at the inlet and outlet of the heat exchanger), (2) the oil upper layer, and (3) the top of the magnetic core. The main reasons for such a choice are, beside the obvious physical differences, their property to be easily measured or determined experimentally and the possibility to evaluate or estimate the hot spot temperature. In this connection, the other oil temperatures are also needed (at the bottom, and in the middle). Generally speaking, any three temperatures may be used, e.g. the middle oil temperature instaed of the upper layer temperature, and/or the hot spot temperature or mean temperature of the winding. Today the hot spot temperature can be promptly measured by means of fibers optics techniques.

(B) The interchange of heat between the bodies by convection and/or conduction are represented by thermal conductances or resistances, either dependent or independent on the temperature; the second option could be applied when a narrow band of temperature is dealt with. When the interchange of heat is negligible, the corresponding thermal conductance may be taken as zero. This treatment results in a much simpler configuration of the heat network (two nodes).

(C) The more or less complicated thermal circuit has its equivalent or analogy in an electric circuit, having C- and R-components, the last being voltage-dependent, and two current sources representing the copper and iron power losses. The circuit may be solved by means of any standard method of Electric Circuit Theory which takes into account the dependence of resistances on voltage situation in the circuit.

5 The equivalent circuit and its further simplifications

The complete equivalent circuit, basd on the one-dimensional heat transfer and valid for non-stationary temperatures in a given narrow temperature range, is show in Fig. 2, in accordance with [6].

Nodes 1–5 relate to the following transformer parts: *1* and *2* primary and secondary windings, *3* iron core, *4* oil and *5* tank.

There are three heat power sources: the primary copper loss P_{Cu1}, the secondary copper loss P_{Cu2}, and the iron loss. The heat transfer takes place, owing to the geometrical disposition and paths for oil circulation, between the following items: the primary and the secondary windings (Λ_1), between each of the windings and the

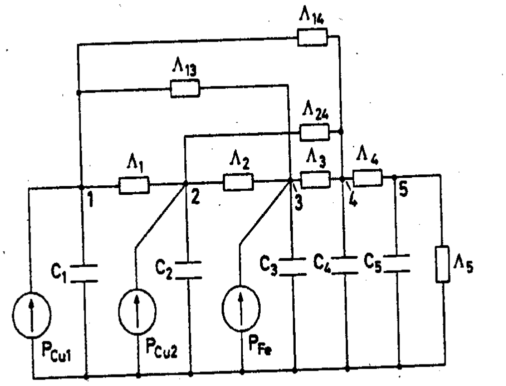

Fig. 2. Equivalent circuit of a transformer with five nodes

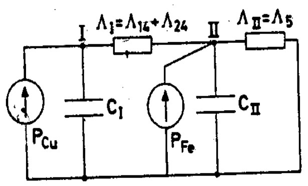

Fig. 3. Equivalent circuit of a transformer with two nodes

core (Λ_{13} and Λ_2), between the core and the oil (Λ_3), between both windings and the oil (Λ_{14} and Λ_{24}), between the oil and tank walls (Λ_4) and finally from the outer surface to the ambient space (Λ_5). Every individual part has its own thermal capacitance ($C_1 - C_5$).

Although significant simplifications have already been made, the given circuit is rather complicated. The mathematical expressions are clumsy and not easy to apply. It is therefore a natural matter to tend to further simplify the circuit. The approach, which is going to be exposed, is – up to some extent – similar to that given in [4]. The first step could be putting both windings together. In such a way, the order of the characteristic equation is reduced by one. This treatment can be justified by the fact that all properly designed transformers in a good state have all important factors in both the primary and secondary windings nearly equal in every instant. These factors are: copper losses, masses and consequently the temperature rises. It is also logical to combine the thermal capacitance of the tank with that of the oil, owing to the fact that its value is relatively small whereas the corresponding conductance Λ_4 is sufficiently large. On the other hand, owing to physical processes in the transformer, and taking into account some experimental data from [6], the values of thermal conductances between the winding and the iron core may be considered as negligibly small. After all these simplifications, the circuit of Fig. 2 reduces to a three-node circuit. A further simplification may be introduced by having in mind that the conductance Λ_3 is relatively large. Practically speaking, it means that nodes 4 and 5 may be put together with node 3. So, the resulting approximation of the circuit is obtained (Fig. 3). The order is reduced from three to two.

Injected powers are now $P_{Cu} = P_{Cu1} + P_{Cu2}$ and P_{Fe}, and capacitanes are $C_I = C_1 + C_2$ and $C_{II} = C_3 + C_4 + C_5$.

Finally, if only the over-all heating of the transformer is considered, taking the oil temperature or the outer surface temperature to be representative, the circuit reduces to the first-order simplest network shown in Fig. 1.

The main purpose of the paper is to show how the parameters of the circuits shown in Fig. 3, which is now going to be treated, can be estimated by means of test data.

6 Experiments and evaluations

In this section, a number of tests made on an oil-filled transformer, will be described. They were carried out with the aims:

(1) to determine the parameters of the thermal equivalent circuit; and

(2) to verify the numerical calculations made on the basis of the exposed method.

In this way, the model to be developed will be used as the basis for easy calculations with the purpose to predict (or at least to estimate) critical temperatures in a transformer for any typical transient state.

6.1 Description of the experiments

The experiments were carried out on a three-phase transformer with following ratings: $S_n = 16\,kVA$, Dzn connection, 6 kV/0.4 kV, $u_k = 6.5\,\%$, $P_{Fen} = 160\,W$ and $P_{Cun} = 740\,W$ (hence $P_{dn} = P_{Cun} + P_{Fen} = 900\,W$). The temperatures were measured: at the upper oil layer, at the inlet and the outlet of the heat exchanger; and the mean winding temperature was obtained by the increase-of-resistance method. All remaining temperatures were measured by thermocouples. The complete measurement was automatized, the obtained values were treated by means of specially designed devices including PC's with specialized program packages. Details of the measurements and acquisitions of the data are published in [9]. As the result, the temperatures as function of time are obtained.

The following tests were made, in respect to time pattern (Fig. 4):

(a) Loading with power loss P_2, after a constant loss P_1 – the transition from one steady state to another, and vice versa.

(b) Overloading having the short-time duration Δt.

(c) Intermittent periodic overloading (after as well as before the quasi-stationary state had been reached), the losses varying regularly between P_2 (of duration t_1) and P_1 (of duration $t_2 - t_1$); the temperature does not reach steady state value.

(d) A typical daily diagram, with about 15 changes of power during 24 hours, the power loss (P_d) varying in accordance with the load (S) variation within the limits [23 % S_n, 151 % S_n].

Due to the non-linear heat transfer, the following values were taken as different in the described series of experiments:

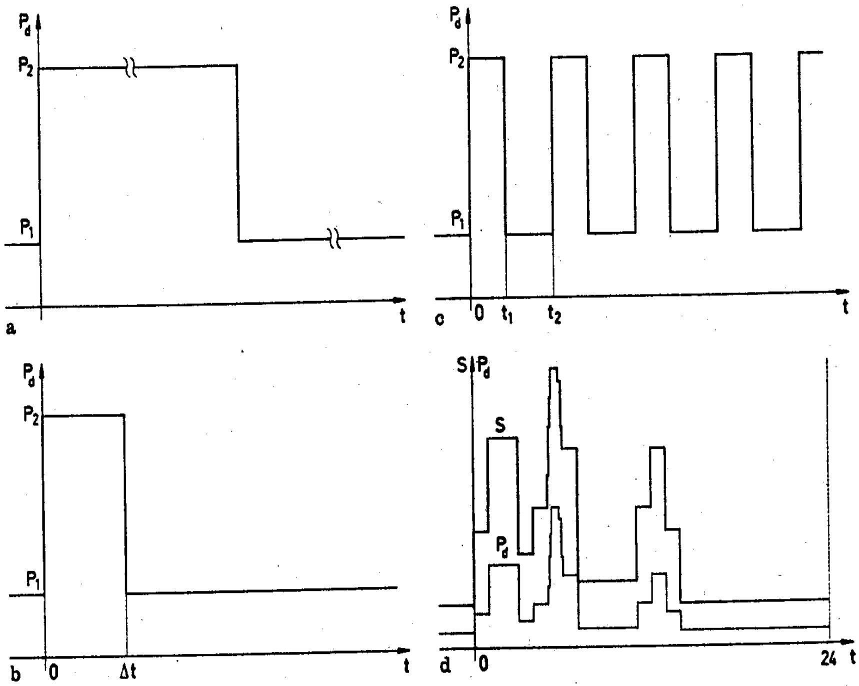

Fig. 4a–d. Time patterns taken for laboratory test. a transition from one steady state to another, b Short-time overloading, c intermittent periodic overloading, d typical daily load diagram

(A) The test (a) for three pairs of $\{P_1, P_2\}$ values.

(B) The test (b) for two pairs $\{P_1, P_2\}$ and two values Δt.

(C) The test (c) for several values of $\{t_1, t_2\}$, in most cases it was taken $t_1 = 0.5\,t_2$.

All experiments were made with short-circuited secondary winding, so that only one heat source existed (P_{Fe} in Fig. 3 is equal zero). Such a procedure makes the tests cheaper and shorter. This restriction is going to be eliminated in the later work by the same authors.

Beside the above tests, where the losses were constant, a set of experiments was undertaken where the current, i. e. the transformer load, is maintained constant. The obtained results did not show any essential difference and are, at least for the time being, discarded.

6.2 *Fitting of experimental curves with analytical ones*

If the temperature curves obtained in the set (a) (for $P_1 = 0$) of the test, are analyzed, it can be concluded that each of them may be fitted with a sufficient accuracy (± 0.8 K) by a curve having the expression

$$\theta = \theta_h^{(1)}\left(1 - e^{-\frac{t}{T_{1h}}}\right) + \theta_h^{(2)}\left(1 - e^{-\frac{t}{T_{2h}}}\right) + \theta_h^{(3)}\left(1 - e^{-\frac{t}{T_{3h}}}\right) \tag{3}$$

for heating up, and the expression

$$\theta = \theta_c^{(1)} e^{-\frac{t}{T_{1c}}} + \theta_c^{(2)} e^{-\frac{t}{T_{2c}}} + \theta_c^{(3)} e^{-\frac{t}{T_{3c}}} \tag{4}$$

for colling down.

The plots of experimental and analytical curves are shown in Figs. 5 and 6 for heating up and cooling down, respectively, when the losses were P_2 110% of the rated value and 61% of the rated value. Figs. 5a and 6a represent the copper temperature and Figs. 5b and 6b the oil temperature. As it is shown in [10], each individual curve may be represented by its own analytical function. In this way, a high accuracy is obtained. In other words, the experimental and the analytical functions coincide almost ideally.

Unfortunately, the time constants T_{ih} and T_{ic} are not equal, showing enormous differences. The same holds for partial temperature rises θ_i. Moreover, all corresponding time constants change also when the power is reduced. The differences for the largest time constants vary up to 2 times, and for the smallest even 6 times [10]. All these variations may be attributed to the well-known dependence of the thermal conductances on temperature. As a consequence, such broad differences of parameters make this approach unpractical.

6.3 *Developing the nonlinear model*

Due to the above mentioned reasons another way of fitting was adopted. The second-order circuit (Fig. 3; P_{Fe} is equal to zero) is taken as the basis, having temperature-dependent conductances in accordance with the well-known functional laws

$$\Lambda_I = \Lambda_{01}(\theta_1 - \theta_2)^{n_1} \tag{5}$$

$$\Lambda_{II} = \Lambda_{02}\theta_2^{n_2} \tag{6}$$

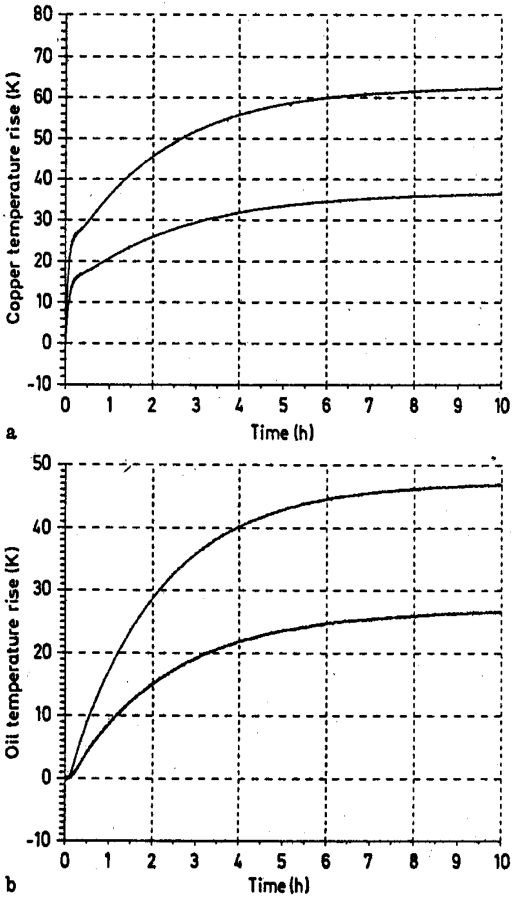

Fig. 5a, b. Temperatures as responses to the step function of power losses (1.1 P_{dn} and 0.61 P_{dn}). **a** average winding temperatures, **b** oil upper layer temperatures

where θ_1 and θ_2 designate the temperature rises of the points I and II (see Fig. 3). The determination of Λ_{01}, Λ_{02}, n_1 and n_2 for the heating-up processes can be made on the basis of 2 or 3 (for better accuracy) steady states obtained by experiment. The procedure of estimating the C-values is described in the Appendix and in [13]. The algorithm of estimating all parameters (for two conductances and two capacitances) is shown. The parameters were estimated on the basis of measured temperature curves when the transformer was being loaded by a step function of copper losses equal to 110% of total rated losses. Figure 7 represents the measured and calculated temperatures, as well as their difference, for the value of losses equal to those in the test for capacitance estimation (Figs. 7a and 7b), and in the test when the loss was 61% of the rated value (Figs. 7c and 7d). The accuracy achieved is fairly well, although it is much lower than in the preceding case; one may conclude that the second-order circuit with temperature-dependent conductances is quite satisfactory if the accuracy requirements are not too rigorous. The drawback of the method is the fact, that no analytic functions exist like those in the former case (Eqs. 3 and 4). Therefore, for the analysis and/or numerical investigation, the computer should be available, which is today a minor request.

To obtain a proper accuracy, it turned out to be necessary to choose rather different parameters in expressions (5) and (6) when the thermal process follows the decreasing of a transformer load, than in the case of increasing it. This is probably due to the fact that the temperature distribution throughout the complicated geometry of the transformer is quite different in these two cases, a fact which is not taken into account in the very simple equivalent circuit used here. Fortunately, it is sufficient to take new values only for the thermal conductances, leaving the values of all capacitances equal to those obtained in the heating-up test. The new estimation was carried out on the basis of the cooling-down test beginning from the steady state with 110% of full rated losses and ending with the state corresponding to 61% of rated value.

It is interesting to note that it seems that only two, or maximum three, sets of values (see Table 1), one for heating-up, the second for cooling-down and – eventually – the third for very low loads (which will be discussed in section 6.5), are necessary to obtain very good accuracy. These values stay valid also when severe or intermittent loads with arbitrary duration are investigated.

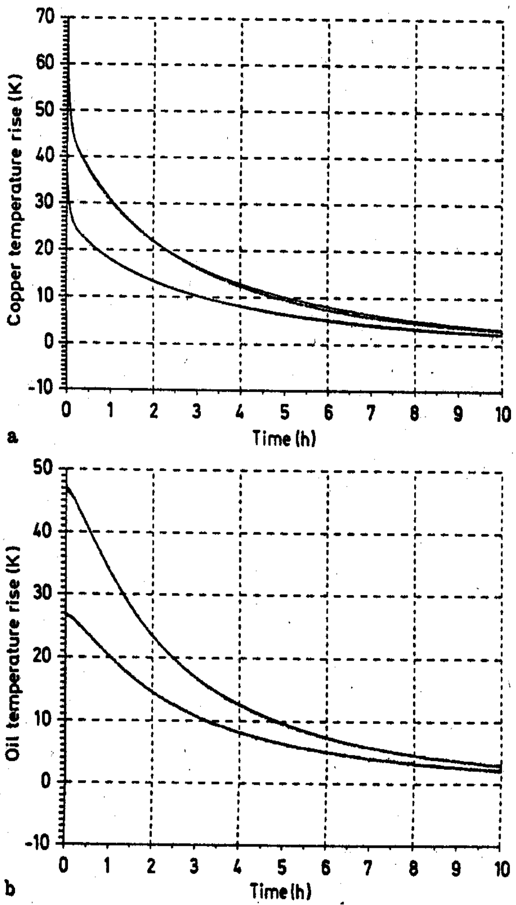

Fig. 6a, b. Temperatures during cooling-down beginning from the steady state reached with power losses 1.1 P_{dn} and 0.61 P_{dn}. **a** average winding temperatures, **b** oil upper layer temperatures

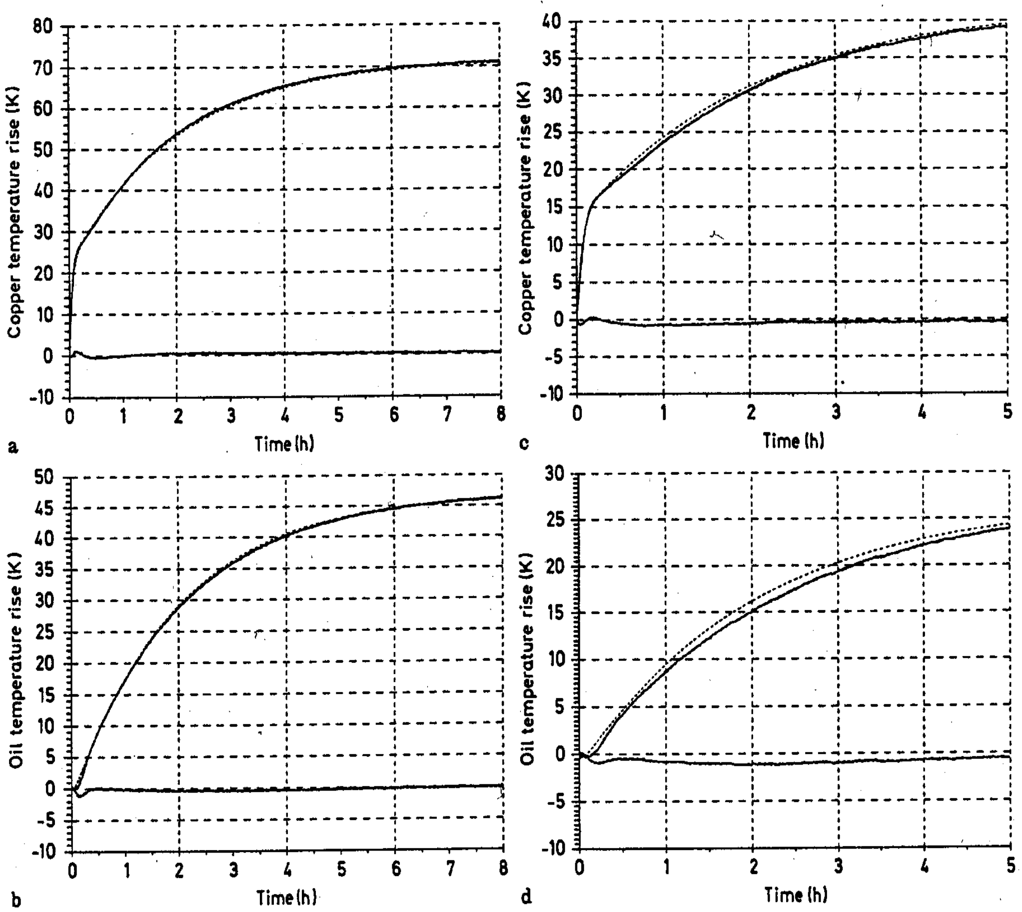

Fig. 7a–d. Temperatures during heating-up with the step function of power losses for parameter estimation. a upper winding part for 1.1 P_{dn}, b oil upper layer for 1.1 P_{dn}, c upper winding part 0.61 P_{dn}, d oil upper layer for 0.61 P_{dn}

Table 1. Circuit parameters on the basis of one heating-up and two cooling down tests. In the last test, the loss power was reduced from the read value (P_{dn}) down to 2/3 P_{dn}, and alternatively to the complete transformer shut-off. In both cases the initial state was the state reached with rated losses

	$\Lambda_I(W/K)$		$\Lambda_{II}(W/K)$		C_I	C_{II}
	Λ_{01}	n_1	Λ_{02}	n_2	(kJ/K)	(kJ/K)
$\Delta P > 0$	18.16	0.25	18.10	0.039	9.416	145.6
$\Delta P < 0$ $P_2 > 0$	12.07	0.350	5.673	0.349	9.416	145.6
$\Delta P < 0$ $P_2 = 0$	15.94	0.5177	8.071	0.2556	23.22	162.5

The procedure for estimating Λ_{01}, Λ_{02}, n_1 and n_2 is a particular case incorporated in the (general) algorithm for all the parameters, given in the Appendix and in [13].

The circuit with parameters established in this way was tested on loadings with short duration (Fig. 4b), periodic loading (Fig. 4c) and a typical daily diagram (Fig. 4d) – Figs. 8, 9 and 10. As mentioned before, the congruence was almost perfect, the absolute temperature error being within 2.7 K for a typical daily diagram.

6.4 Small temperature variations

When the temperatures do not change extensively, and the time intervals observed are not too long, it is not necessary to take into account the dependence of Λ on temperature. This will be shown now by the case of an intermittent duty cycle represented in Fig. 4c $P_1 = 250$ W, $P_2 = 1250$ W, $t_1 = 30$ min, and $t_2 = 60$ min. The experimentally obtained oscillations of copper temperature, as well as that of oil, have both half-waves quite symmetrical. This confirms the assumption that the circuit is linear, i.e. all parameters may be considered independent of temperature, at least for the temperature range in question. If this is true, phasor calculus and Fourier series are permitted for analyzing. In the case of Figs. 9c and 9d, first the dc component of the temperature was calculated, now taking into account the temperature dependence of conductances: afterwards, for the ac component, the customary procedures of linear electrical circuits are applied.

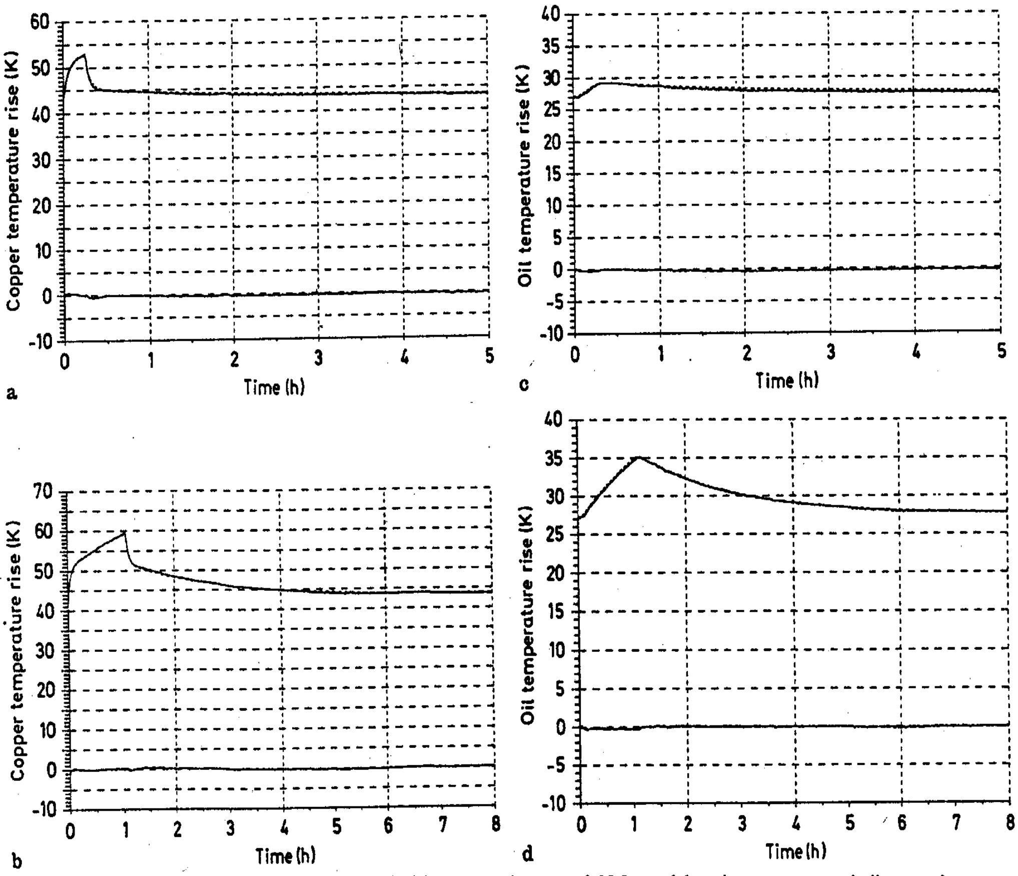

Fig. 8a–d. Temperatures for short term overload with $P_2 = 1.1\,P_{dn}$, $P_1 = 0.55\,P_{dn}$ and duration Δt. a upper winding part for $\Delta t = 15$ min, b oil upper layer for $\Delta t = 15$ min, c upper winding part for $\Delta t = 63$ min, d oil upper layer for $\Delta t = 63$ min

6.5 Prototype tests for data necessary to calculate the circuit parameters

On the basis of the preceding text the authors propose to make three tests on the transformer in order to establish the second-order circuit's parameters. All of these tests represent the temperature response measurements when the power loss takes positive and negative step changes, in accordance with Fig. 11: The first step from zero up to rated loss, the second from this value down to approximately 2/3 of rated loss, and the third from 2/3 of rated loss to zero. On the basis of the obtained curve, and using the algorithm exposed in the Appendix and in [13], all parameters can be obtained. It is interesting to note that in the third set of parameters the capacitances were also changed, because satisfying results could not be reached if the preceding values were used. So, all six parameters were included into the estimation process. Nevertheless, this set of parameters (cooling-down of the shut-off transformer) has not a significant practical meaning, because the second parameter set guarantees very accurate results when the load is at least 20% of the rated value. Therefore, from the practical point of view, two parameter sets suffice. It follows that only three tests have to be made: two heating-up to steady states (or even only one if all of six parameters are subject to the estimation), and one cooling-down test (Fig. 12).

6.6 Summary

As the conclusion of this section, it may be said:

- if heating-up and cooling-down curves, beginning from one steady state to another, are considered, each of them may be represented by a sum of three exponential functions; their magnitude is different when the stationary temperatures are different and when the temperature time change takes the opposite direction;
- if it is desired to have a unique circuit, which seems quite natural, the parameters Λ ought to be taken as temperature dependent, resulting in the need for having a PC for analysis and computation; the best results are obtained if different powers of θ are taken for positive and negative power loss changes in time.

7 Conclusions and prospect for future work

The most often used simplified model of transformer thermal processes, having a single time constant (either with constant or with temperature-depending parameters), is not always feasible. It appears that in most cases the equivalent circuit with at least two nodes should be applied.

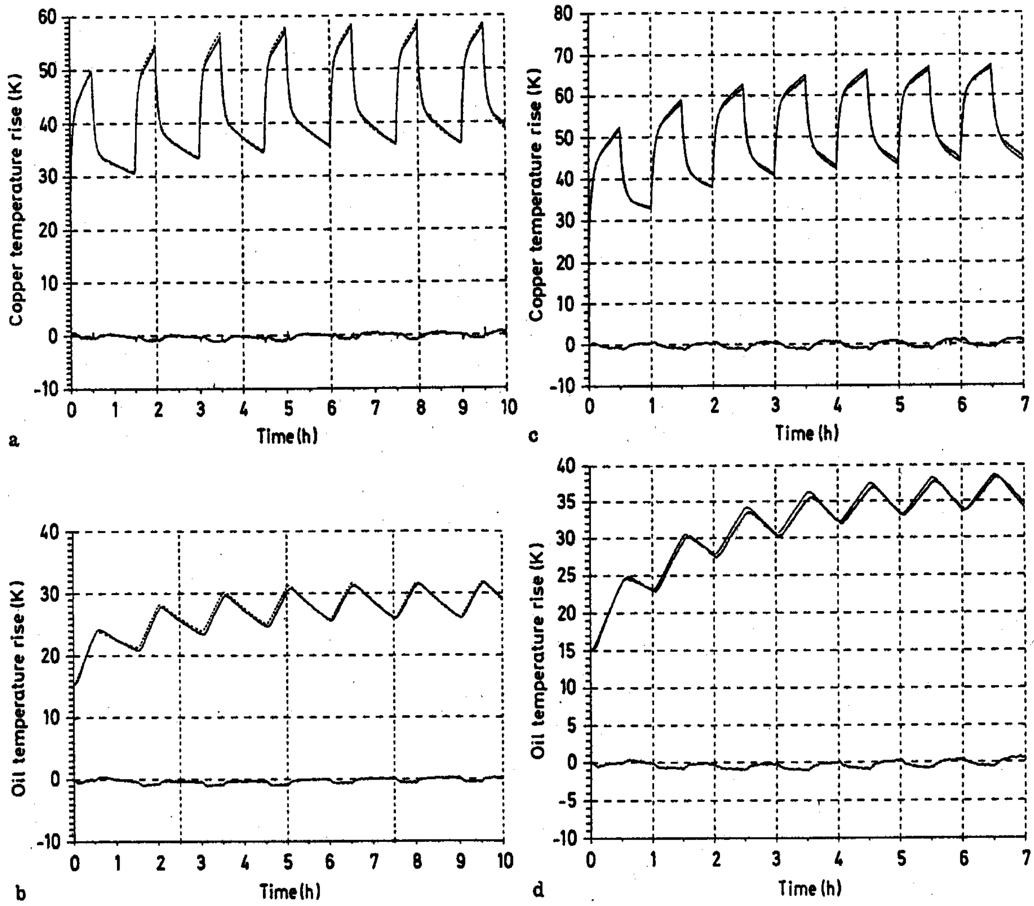

Fig. 9a–d. Temperatures for cyclically varying load with $P_1 = 0.278\,P_{dn}$ and $t_1 = 30$ min. **a** upper winding part for $P_2 = 1.28\,P_{dn}$ and $t_2 = 90$ min, **b** oil upper layer for $P_2 = 1.28\,P_{dn}$ and $t_2 = 90$ min, **c** upper winding part for $P_2 = 1.39\,P_{dn}$ and $t_2 = 60$ min, **d** oil upper layer for $P_2 = 1.39\,P_{dn}$ and $t_2 = 60$ min

This set of equivalent circuits may have a broad application. The most important problem can be the evaluation of critical temperatures of windings and the oil, assuming a certain pattern of the load/time diagram. Such a diagram may be simplified to a great extent (as it used in [2]), or more or less close to real one. This is why the authors feel that the standpoint and investigations exposed here may serve eventually as the basis for the revision of the IEC standard for loading the power transformers [2, 3, 7]. The proposed procedure allows to form the tables given in the Recommendations [2, 7] with much better accuracy, relating to a specified transformer. Moreover, it is also possible to obtain a parameter set (shown in the form of a table) which makes possible the easy determination of temperature when the load has any arbitrary pattern in value and time. Such a table could be composed by a simulation on the basis of the circuit developed in this paper, combined with the time constant determination given in [10].

Concepts and procedures exposed in this paper are confirmed experimentally on a single, relatively small transformer. In order to obtain a more complete verification, further tests should be made, including larger transformers and real operating conditions – with iron losses in the proper place.

Provided that these tests confirm the validity of the proposed modelling of the thermal states of power oil transformers, steps may be taken to revise the corresponding international standards [2, 3, 7]. Instaed of numerous tables, incorporating a single overload during a day, having a constant value – which is far from real daily diagrammes – an algorithm developed on the basis given here can be used, which makes possible the prediction of critical temperatures for overload with arbitrary time pattern. In this way, the transformer can be utilized up to the permissible limits, with no excessive insulation aging. For a given daily load diagram, the aging of transformer insulation could be estimated much more accurately than by means of the method in [2]. The procedure may be used to check the critical temperatures in emengency states.

Appendix

The method of parameter estimation will be explained first by the simplest example illustrated by Fig. 1, and afterwards by the more general case of the forms of circuits used later on in the paper.

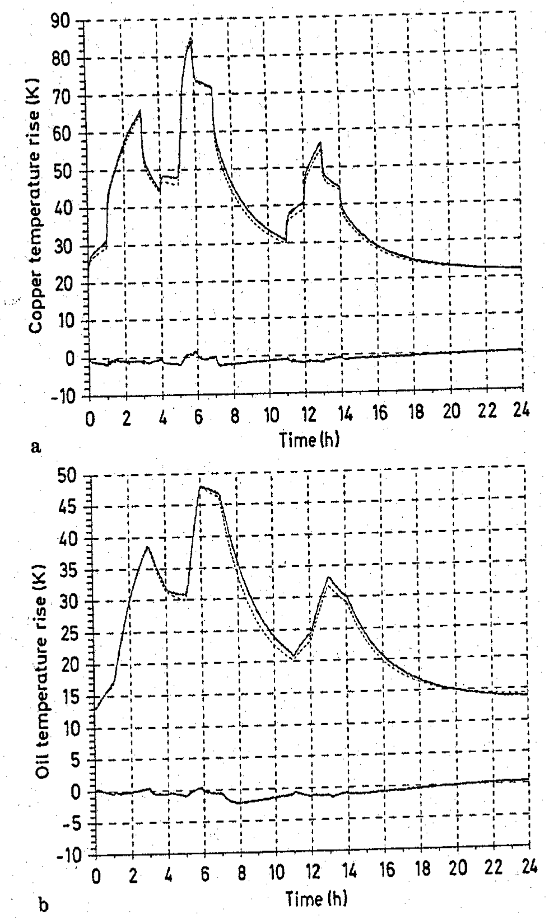

Fig. 10 a, b. Temperatures for the typical daily load diagram of the transformer shown in Fig. 4 d. **a** upper winding part, **b** oil upper layer

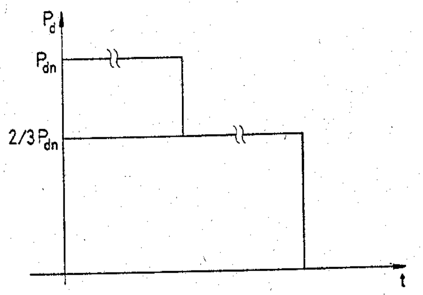

Fig. 11. Test for obtaining parameters relevant to the simulation of any abitrary loading

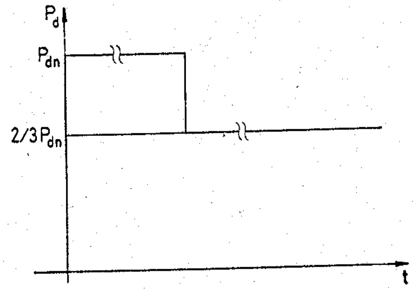

Fig. 12. Test for obtaining parameters relevant to the simulation of realistic loading

The problem is to determine a number of parameters, which cannot be measured directly. As starting data, measured values of characteristic quantities, such as temperatures, are used. The measured values are assumed to be taken in regular time intervals and that the accuracy is always the same.

The relation between quantities which are not measurable, and those which are measured (see Fig. 1) is given by the non-homogeneous and non-linear differential equation:

$$C\frac{d\theta}{dt} + \Lambda\theta = P. \qquad (7)$$

It is convenient to introduce a new parameter k because the time intervals of measurements are always equal:

$$k = \frac{\Delta t}{C}. \qquad (8)$$

The following difference equation results from Eq. (7):

$$\theta_{i+1} - \theta_i = -k\Lambda_i\theta_i + kP_i. \qquad (9)$$

The most interesting case is when the conductance Λ is a function of the temperature, whereas the capacitance is independent of the temperature. Let us suppose that $m+1$ parameters $(a_0, a_1, \ldots, a_m)$ take part in the functional dependence of Λ with temperature.

On the other hand, if n measured values are available, one is able to establish a system of n equations, which represents, in fact, Eq. (9) written for each time interval in a modified and condensed form:

$$f_i(k, a_0, \ldots, a_m, \theta_i, \theta_{i+1}) = 0 \quad i = 0, 1, \ldots, n-1. \qquad (10)$$

System (10) would have a unique solution for $k, a_0, \ldots, a_m$, only if the number of equations were equal to the number of unknown parameters. But usually the number of measurements is much larger, in order to obtain a good accuracy, and consequently the number of equations is larger than the number of parameters to be determined. It follows that all the measured values θ_i^r, $i = 0, \ldots, n$, are to be corrected for a certain amount $\Delta\theta_i$ in order to satisfy the system (10).

With the aim to minimize the difference between the estimated and measured values, one should correct the model parameters $(k, a_0, \ldots, a_m)$ for a certain amount $(\Delta k, \Delta a_0, \ldots, \Delta a_m)$. System (10) may now be written in the form $(i = 0, \ldots, n-1)$:

$$f_i(k + \Delta k, \quad a_0 + \Delta a_0, \ldots, a_m + \Delta a_m,$$
$$\theta_i^r + \Delta\theta_i, \quad \theta_{i+1}^r + \Delta\theta_{i+1}) = 0. \qquad (11)$$

By developing the left sides of all equations into Taylor series in the vicinity of the point representing the assumed solution $(k^0, a_0^0, \ldots, a_m^0)$, and neglecting all higher-order terms, the following system of n equations is obtained:

$$f_i^0 + \sum_{j=0}^{m} \left(\frac{\partial f_i}{\partial a_j}\right)_0 \Delta a_j + \left(\frac{\partial f_i}{\partial k}\right)_0 \Delta k + \left(\frac{\partial f_i}{\partial \theta_i}\right)_0 \Delta\theta_i + \left(\frac{\partial f_i}{\partial \theta_{i+1}}\right)_0 \Delta\theta_{i+1} = 0 \tag{12}$$

where $f_i^0 = f_i(k^0, a_0^0, \ldots, a_m^0)$, and the index 0 at the derivative designates that coefficients $k^0, a_0^0, \ldots, a_m^0$ are to be applied after the differentiation. By assuming a measure of deviation

$$\left(\frac{\partial f_i}{\partial \theta_i}\right)_0 \Delta\theta_i + \left(\frac{\partial f_i}{\partial \theta_{i+1}}\right)_0 \Delta\theta_{i+1} = -\delta_i; \quad i = 0, \ldots, n-1 \tag{13}$$

expression (12), being the error equation system, becomes:

$$f_i^0 + \sum_{j=0}^{m} \left(\frac{\partial f_i}{\partial a_j}\right)_0 \Delta a_j + \left(\frac{\partial f_i}{\partial k}\right)_0 \Delta k = \delta_i; \quad i = 0, \ldots, n-1. \tag{14}$$

By inserting the partial derivatives of the assumed solution into system (14), the following system of equations is obtained

$$\delta_i = f_i^0 - h_i^0 \Delta k - \sum_{j=0}^{m} g_{j,i}^0 \Delta a_j; \quad i = 0, \ldots, n-1 \tag{15}$$

$$\Lambda_i = \begin{bmatrix} \Lambda_{1i}(a_1, \ldots, a_{m1}) & 0 & \ldots & 0 \\ 0 & \Lambda_{2i}(a_{m1+1}, \ldots, a_{m1+m2}) & \ldots & 0 \\ \vdots & & & \\ 0 & 0 & \ldots & \Lambda_{gi}(a_{x-mg+1}, \ldots, a_x) \end{bmatrix} \tag{21}$$

$$\left(\frac{\partial f_i}{\partial k}\right)_0 = -h_i^0 \quad i = 0, \ldots, n-1 \tag{16}$$

$$\left(\frac{\partial f_i}{\partial a_j}\right)_0 = -g_{j,i}^0; \quad j = 0, \ldots, m. \tag{17}$$

Increments $\Delta k, \Delta a_0, \ldots, \Delta a_m$ are obtained when the function

$$J = \sum_{i=0}^{n-1} \delta_i^2 \tag{18}$$

is minimized.

By puting all derivatives of J with respect to the increments equal to zero, the following equation system is obtained

$$\sum_{i=0}^{n-1} (\delta_i h_i^0) = 0$$
$$\sum_{i=0}^{n-1} (\delta_i g_{j,i}^0) = 0 \quad j = 0, 1, \ldots, m \tag{19}$$

From Eqs. (19), the unknown $m+2$ increments $(\Delta k, \Delta a_0, \ldots, \Delta a_m)$ are obtained.

If the largest increment of each parameter has an absolute value smaller than a given small number, representing the criterion of the approximation quality, the iteration procedure is finished. If the opposite is true, a new set of starting solutions is determined and the procedure of searching for the least square of deviations is repeated $(k = k^0 + \Delta k, a_0 = a_0^0 + \Delta a_0, \ldots, a_m = a_m^0 + \Delta a_m)$.

The exposed methods is applied to the thermal circuits used throughout this paper. Referring to node analyses method, known from the Electric Circuit Theory, a matrix difference equation may be established in the form

$$\boldsymbol{\theta}_{i+1} = \{\mathbf{I} - \mathbf{C}^{-1}\Delta t \boldsymbol{\eta}_i\} \boldsymbol{\theta}_i + \mathbf{C}^{-1}\Delta t \mathbf{P}_i. \tag{20}$$

Here, $\boldsymbol{\eta}_i$ is the node analyse matrix $\boldsymbol{\eta}_i = \mathbf{A}\boldsymbol{\Lambda}_i\mathbf{A}^T$($\mathbf{A}$ is the incident matrix of the network having all capacitances and sources omitted, and $\boldsymbol{\Lambda}_i$ the diagonal matrix of branch conductances).

$\mathbf{C}$ is a diagonal matrix; it follows that its inverse matrix $\mathbf{K}$ is also diagonal. The following designations of its elements are applied: $k_{jj} = \Delta t / C_{jj}; j = 1, \ldots, c$ (c being the number of nodes in the network).

The parameter functions of conductances are functions of measured values, the number of parameter being different for specific functions. The diagonal matrix $(g \times g)$ of conductances is

The total number of conductance functions is equal to the number of branches (g) in the network (without the capacitance and generator branches), the number of unknown parameters of conductance functions being equal to the number of the sum of unknown parameters belonging to these function.

In an analogue way, illustrated by Eqs. (4)–(8), the following matrix equation system is obtained

$$\boldsymbol{\delta}_i = \mathbf{F}_i^0 - \mathbf{H}_i^0 \Delta\mathbf{K} + \mathbf{K}^0 \Delta\boldsymbol{\eta}_i \boldsymbol{\theta}_i'; \quad i = 0, \ldots, n-1 \tag{22}$$

where $\Delta\mathbf{K}$ is a vector with size $(c \times 1)$, and $\mathbf{H}_i^0$ is a diagonal matrix with the size $(c \times c)$:

$$\mathbf{H}_i^0 = -\mathbf{I}\{\mathbf{A}\boldsymbol{\Lambda}_i^0\mathbf{A}^T\boldsymbol{\theta}_i' - \mathbf{P}_i\}; \quad i = 0, \ldots, n-1 \tag{23}$$

where $\mathbf{I}$ designates the unity matrix.

Let us designate the product $\mathbf{A}^T\boldsymbol{\theta}_i'$ in the particular term of the equation (22)

$$\mathbf{K}^0 \Delta\boldsymbol{\eta}_i \boldsymbol{\theta}_i' = \mathbf{K}^0 \mathbf{A} \Delta\boldsymbol{\Lambda}_i \mathbf{A}^T \boldsymbol{\theta}_i' \tag{24}$$

as

$$\mathbf{V}_i = \mathbf{A}^T\boldsymbol{\theta}_i^r = \begin{bmatrix} v_{1i} \\ v_{2i} \\ \cdots \\ v_{gi} \end{bmatrix}. \qquad (25)$$

In order to obtain a form suitable for determining parameter increments, the generating of matrices $\Delta\Lambda_i'(g \times x)$ and $\mathbf{V}_{iv}'(x \times 1)$ is made from matrices $\Delta\Lambda_i$ and $\mathbf{V}_i$.

$$\Delta\Lambda_1' = \begin{bmatrix} \left(\frac{\partial A_1}{\partial a_1}\right)_i^0 \Delta a_1 \ldots \left(\frac{\partial A_1}{\partial a_{m1}}\right)_i^0 \Delta a_{m1} & \ldots & 0 \\ \vdots & & \\ 0 & \ldots & \left(\frac{\partial A_g}{\partial a_{x-mg+1}}\right)_i^0 \Delta a_{x-mg+1} \ldots \left(\frac{\partial A_g}{\partial a_x}\right)_i^0 \Delta a_x \end{bmatrix} \qquad (26)$$

$$\mathbf{V}_{iv}'^T = [v_{1i} \ldots v_{1i} v_{2i} \ldots v_{2i} \ldots v_{gi} \ldots v_{gi}]. \qquad (27)$$

The matrix $\Delta\Lambda_i'$ may be represented as the product of two matrices $\partial\Lambda_i^0$ and $\Delta\boldsymbol{\alpha}$:

$$\partial\Lambda_i^0 = \begin{bmatrix} \left(\frac{\partial A_1}{\partial a_1}\right)_i^0 \ldots \left(\frac{\partial A_1}{\partial a_{m1}}\right)_i^0 & \ldots & 0 \\ \vdots & & \\ 0 & \ldots & \left(\frac{\partial A_g}{\partial a_{x-mg+1}}\right)_i^0 \ldots \left(\frac{\partial A_g}{\partial a_x}\right)_i^0 \end{bmatrix} \qquad (28)$$

$$\Delta\boldsymbol{\alpha} = \begin{bmatrix} \Delta a_1 & \ldots & 0 \\ & \ddots & \\ 0 & \ldots & \Delta a_x \end{bmatrix} \qquad (29)$$

Eq. (22) is now:

$$\boldsymbol{\delta}_i = \mathbf{F}_i^0 - \mathbf{H}_i^0\Delta\mathbf{K} + \mathbf{K}^0\mathbf{A}\,\partial\Lambda_i^0\mathbf{V}_i'\Delta\mathbf{a}; \quad i = 0, \ldots, n-1 \qquad (30)$$

where $\Delta\mathbf{a}$ represents a vector having the elements of the matrix $\Delta\boldsymbol{\alpha}$ at the principal diagonal and $\mathbf{V}_i'$ the diagonal matrix with the elements of the vector $\mathbf{V}_{iv}'$.

With the matrix product

$$\boldsymbol{\Gamma}_i^0 = -\mathbf{K}^0\mathbf{A}\,\partial\Lambda_i^0\mathbf{V}_i' \qquad (31)$$

the minimization of the function

$$J = \sum_{i=0}^{n-1}\sum_{j=1}^{c} \delta_{ji}^2 \qquad (32)$$

with respect to increments $\Delta k_1, \ldots, \Delta k_c, \Delta a_1, \ldots, \Delta a_x$ provides the following system of equations:

$$\sum_{i=0}^{n-1} \mathbf{H}_i^0\boldsymbol{\delta}_i = 0$$
$$\sum_{i=0}^{n-1} \boldsymbol{\Gamma}_i^{0T}\boldsymbol{\delta}_i = 0. \qquad (33)$$

By putting (22) in (33), the block-matrix equation is obtained

$$\begin{bmatrix} \mathbf{R} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{T} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{K} \\ \Delta\mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{II}_k \\ \mathbf{II}_a \end{bmatrix} \qquad (34)$$

where

$$\mathbf{R} = \sum_{i=0}^{n-1} \mathbf{H}_i^0\mathbf{H}_i^0; \quad \mathbf{S} = \sum_{i=0}^{n-1} \mathbf{H}_i^0\boldsymbol{\Gamma}_i^0; \quad \mathbf{T} = \sum_{i=0}^{n-1} \boldsymbol{\Gamma}_i^{0T}\boldsymbol{\Gamma}_i^0 \qquad (35)$$
$$\mathbf{II}_k = \sum_{i=0}^{n-1} \mathbf{H}_i^0\mathbf{F}_i^0; \quad \mathbf{II}_a = \sum_{i=0}^{n-1} \boldsymbol{\Gamma}_i^{0T}\mathbf{F}_i^0$$

having in mind that

$$\boldsymbol{\Gamma}_i^{0T}\mathbf{H}_i^0 = (\mathbf{H}_i^0\boldsymbol{\Gamma}_i^0)^T = \mathbf{S}^T. \qquad (36)$$

Solving Eqs. (34), i.e. by means of the Gaussian algorithm, the unknown parameters $\Delta k_j, j = 1, \ldots, c$ and $\Delta a_l, l = 1, \ldots, x$ are obtained. The criterion for the exit from the iteration loop and going over to the next parameter calculation is the same as in the preceding case of a single differential equation.

In order to make the procedure applied to the higher-order circuits more clear, the following example concerning a second-order circuit is given. It is shown in Fig. 3 and has the temperature dependences of Λ given by Eqs. (5) and (6). Matrices occurring in (20) have now the values

$$\mathbf{C} = \begin{bmatrix} C_1 & 0 \\ 0 & C_{II} \end{bmatrix} \tag{37}$$

$$\partial\Lambda_i{}^0 = \begin{bmatrix} (\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0} & \lambda^0_{01}(\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0} \ln(\theta^r_{1,i} - \theta^r_{2,i}) & 0 & 0 \\ 0 & 0 & (\theta^r_{2,i})^{n_1{}^0} & \lambda^0_{02}(\theta^r_{2,i})^{n_2{}^0} \ln \theta^r_{2,i} \end{bmatrix}. \tag{53}$$

$$\mathbf{P}_i = \begin{bmatrix} P_{Cui} \\ P_{Fei} \end{bmatrix} \tag{38}$$

$$\boldsymbol{\theta}_i = \begin{bmatrix} \theta_{1,i} \\ \theta_{2,i} \end{bmatrix} \tag{39}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \tag{40}$$

$$\Lambda_i = \begin{bmatrix} \Lambda_{I,i} & 0 \\ 0 & \Lambda_{II,i} \end{bmatrix} = \begin{bmatrix} \lambda_{01}(\theta_{1,i} - \theta_{2,i})^{n_1} & 0 \\ 0 & \lambda_{02}\theta^{n_2}_{2,i} \end{bmatrix} \tag{41}$$

$$\boldsymbol{\eta}_i = \begin{bmatrix} \Lambda_{I,i} & -\Lambda_{I,i} \\ -\Lambda_{I,i} & \Lambda_{I,i} + \Lambda_{II,i} \end{bmatrix} \tag{42}$$

and those in the expression (22):

$$\mathbf{F}_i{}^0 = \boldsymbol{\theta}^r_{i+1} - \{\mathbf{I} - \mathbf{K}^0\Delta t\boldsymbol{\eta}_i{}^0\}\,\boldsymbol{\theta}_i{}' - \mathbf{K}^0\Delta t\mathbf{P}_i \tag{43}$$

$$\mathbf{K} = \begin{bmatrix} k_I & \\ & k_{II} \end{bmatrix}; \quad k_I = \frac{\Delta t}{C_I}, \quad k_{II} = \frac{\Delta t}{C_{II}} \tag{44}$$

$$\Delta\Lambda_i = \begin{bmatrix} \Delta\Lambda_{I,i} & 0 \\ 0 & \Delta\Lambda_{II,i} \end{bmatrix} \tag{45}$$

$$\Delta\boldsymbol{\eta}_i = \begin{bmatrix} \Delta\Lambda_{I,i} & -\Delta\Lambda_{I,i} \\ -\Delta\Lambda_{I,i} & \Delta\Lambda_{I,i} + \Delta\Lambda_{II,i} \end{bmatrix} \tag{46}$$

where the differentials of conductivity functions are

$$\begin{aligned} \Delta\Lambda_{I,i} &= (\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0}\,\Delta\lambda_{01} \\ &\quad + \lambda^0_{01}(\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0} \ln(\theta^r_{1,i} - \theta^r_{2,i})\,\Delta n_1 \\ \Delta\Lambda_{II,i} &= (\theta^r_{2,i})^{n_2{}^0}\,\Delta\lambda_{02} + \lambda^0_{02}(\theta^r_{2,i})^{n_2{}^0} \ln \theta^r_{2,i}\Delta n_2 \end{aligned} \tag{47}$$

The vector $\mathbf{V}_i$ in this case becomes

$$\mathbf{V}_i{}^T = [\theta^r_{1,i} - \theta^r_{2,i} \quad \theta^r_{2,i}] \tag{48}$$

and the new matrix $\Delta\Lambda_i'$ and the rector $\mathbf{V}'_{iv}$ are:

$$\Delta\Lambda_i' = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ 0 & 0 & m_{23} & m_{33} \end{bmatrix} \tag{49}$$

where

$$\begin{aligned} m_{11} &= (\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0}\,\Delta\lambda_{01} \\ m_{12} &= \lambda^0_{01}(\theta^r_{1,i} - \theta^r_{2,i})^{n_1{}^0} \ln(\theta^r_{1,i} - \theta^r_{2,i})\,\Delta n_1 \\ m_{23} &= (\theta^r_{2,i})^{n_2{}^0}\,\Delta\lambda_{02} \\ m_{33} &= \lambda^0_{02}(\theta^r_{2,i})^{n_2{}^0} \ln\theta^r_{2,i}\,\Delta n_2 \end{aligned} \tag{50}$$

$$\mathbf{V}'^T_{iv} = [\theta^r_{1,i} - \theta^r_{2,i} \quad \theta^r_{1,i} - \theta^r_{2,i} \quad \theta^r_{2,i} \quad \theta^r_{2,i}]. \tag{51}$$

The component matrices of matrix $\Delta\Lambda_i'$ are:

$$\Delta\boldsymbol{\alpha} = \begin{bmatrix} \Delta\lambda_{01} & 0 & 0 & 0 \\ 0 & \Delta n_1 & 0 & 0 \\ 0 & 0 & \Delta\lambda_{02} & 0 \\ 0 & 0 & 0 & \Delta n_2 \end{bmatrix} \tag{52}$$

The remaining matrices are simply obtained by multiplying matrices given in (37)–(53), applying formulae from the first part of the Appendix.

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